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Stacking of Controlled Source Electrical Geophysical Data

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Stacking of Controlled Source Electrical Geophysical Data

Robust Stacking Filters and Suggested Processing Schemes

Overview

Stacking of periodic controlled source electrical geophysical data is a subject that is often taken too lightly. The most commonly used stacking algorithm suffers substantial disadvantages, which in many realistic and frequently encountered conditions are catastrophic to data quality.

A robust approach to stacking issues ...

- a) employs optimized half-period weights to reject noise - including linear drift;
- b) recognizes the differences between airborne and ground operations, or low-frequency (IP) vs. higher frequency (EM and CSAMT) operations; and
- c) tailors the stacking design to those considerations.

This paper points out the severe weaknesses in the most popular stacking algorithm (called normal stacking) and offers designs for greatly improved approaches, which are based on an algorithm designed by Mark Halverson¹ in the mid 1960's. These approaches lead to further innovative processing schemes for moving platform data that entail a very large number of transmitted periods such as is collected in airborne EM.

Introduction

Stacking, as discussed in this paper, is a process of combining or averaging digital representations of a periodic waveform to reduce noise and estimate the signal component. The common presumption in controlled source applications is that the signal of interest is polarity symmetric; i.e. it bears symmetry that implies it is comprised of odd-harmonics of the fundamental frequency only. Stacking of symmetric and periodic controlled-source waveforms therefore implies that all information is contained in a single half-period since the full-period component is comprised of the positive and negative versions of the half-period. Hence, stacking as we discuss herein produces an estimate of the half-period component reflecting the odd-harmonics of a periodic waveform.

What is sometimes overlooked in actual applications is that even if an excitation waveform is not symmetric, that is if it has even-harmonic as well as odd-harmonic components, stacking will properly yield the odd-harmonic response/result if the response system is linear. Hence, a great deal of unnecessary expense frequently goes into building transmitters that ensure a high level of symmetry in their outputs. What may be required is a high level of true periodicity, thus ensuring that there is no energy between the integer harmonics. However, even this requirement can generally be precluded through the use of an accurate [current] monitor of the excitation waveform. The requirement of a sufficient level of response linearity is generally critical.

¹ Chief Research Geophysicist and other capacities, Anaconda Copper (later the Anaconda Company), 1960-1979

Stacking As a Linear Filter

To understand stacking in detail, it is important to realize that stacking as normally implemented is essentially a moving average (MA) or finite impulse response (FIR) linear filtering process. Let us consider the normal stacking process whereby positive and negative half-periods are subtracted from each other. Presuming that, for simplicity's sake, there are 8 samples per half-period, we look at a stack depth of 4 half-periods. The equivalent MA filter representing this stacking operation is:

$$[1/4, 0, 0, 0, 0, 0, 0, 0, -1/4, 0, 0, 0, 0, 0, 0, 0, 1/4, 0, 0, 0, 0, 0, 0, -1/4, 0, 0, 0, 0, 0, 0]$$

More typically, of course, there are many more samples per half-period and so MA stacking filters have many zeros separating the non-zero taps. It is the weights of those non-zero taps, the half-period weights of the whole stack that we may adjust, sometimes in extremely subtle ways, to optimize the characteristics of stacking filters. So to repeat, it is critical to understand and consider the operation of stacking as essentially the application of an MA filter.

An alternative approach to stacking per se, is to simply 1) Fourier transform (using the Discrete Fourier Transform) an ensemble spanning an integer number of periods, 2) zero results for all frequencies except those reflecting the odd harmonics, and 3) inverse transform back to the time-domain. Careful investigation shows that this result is, in fact, exactly the same as normal stacking of the same ensemble length; i.e. there are no additional benefits over normal stacking. While the notion of effectively performing stacking using frequency-domain/time-domain transformations may hold advantages for some special purposes, in general it tends to mask important concerns regarding the bands between the discrete frequencies and can easily produce untoward effects.

Halverson Linear Drift Removal Stacking

While developing induced polarization (IP) technologies at the Anaconda Copper Company in the 1960s, Mark Halverson developed a stacking approach that removes linear drift in data. This was done largely in response to Anaconda's desire to abandon porous pot electrodes in favor of solid (usually metal) electrodes as it developed proprietary large-channel-capacity geophysical systems. Solid electrodes drift in potential for a long time following emplacement and over a period up to many minutes the drift is quite linear. It was recognized also that very low frequency electromagnetic noise can be quasi linear over shorter acquisition times, and thus could also benefit from drift removal. Halverson deduced that linear drift in data is removed when three half-periods are averaged using the following weights:

$$1/4, -1/2, 1/4$$

We may carry this concept forward to allow any stack depth greater than three half-periods, odd or even. Conceptually, one forms Halverson stacking weights for arbitrary stack depths by shifting and summing the basic three half-period unit. Weights for a stack depth of six half-periods are determined as follows:

$$\begin{aligned} & (1/4, -1/2, 1/4)/4 \\ + & (-1/4, 1/2, -1/4)/4 \\ + & (1/4, -1/2, 1/4)/4 \\ + & (-1/4, 1/2, -1/4)/4 \\ = & [1, -3, 4, -4, 3, -1]/16 \end{aligned}$$

The scheme unfolds as follows:

- Weights for a stack depth of 4 half-periods: $[\frac{1}{8}, -\frac{3}{8}, \frac{3}{8}, -\frac{1}{8}]$
 Weights for a stack depth of 5 half-periods: $[\frac{1}{12}, -\frac{3}{12}, \frac{4}{12}, -\frac{3}{12}, \frac{1}{12}]$
 Weights for a stack depth of 6 half-periods: $[\frac{1}{16}, -\frac{3}{16}, \frac{4}{16}, -\frac{4}{16}, \frac{3}{16}, -\frac{1}{16}]$
 Weights for a stack depth of 7 half-periods: $[\frac{1}{20}, -\frac{3}{20}, \frac{4}{20}, -\frac{4}{20}, \frac{4}{20}, -\frac{3}{20}, \frac{1}{20}]$
 Weights for a stack depth of 8 half-periods: $[\frac{1}{24}, -\frac{3}{24}, \frac{4}{24}, -\frac{4}{24}, \frac{4}{24}, -\frac{4}{24}, \frac{3}{24}, -\frac{1}{24}]$

Again, bear in mind that all of these operators remove linear drift in data. This is contrasted with normal stacking wherein all half-periods are weighted equally. Normal stacking does not remove linear drift, which has the effect of adding an offset to the stacked result(s) with magnitude dependent on the drift rate - the more severe the drift the larger the offset in the result². Note that this offset is repeatable in the stacked results; hence normal stacking will misleadingly indicate perfect repeatability in the case of linear drift noise. This is a very serious weakness in many conditions.

The general formula for Halverson stacking half-period weights for stack depths > 4 is:

$$\frac{[1, -3, 4, -4, 4, -4, \dots, 4(-1)^{n_s-3}, 3(-1)^{n_s-2}, 1(-1)^{n_s-1}]}{4(n_s - 2)}, \text{ where } n_s \text{ is the stack depth in half-periods.}$$

So we see in general that unlike normal stacking with uniform weight amplitudes, Halverson stacking has a slight taper at the ends, as shown below in Figure 1. Note that in order to uniformly weight data in a record (to the degree possible), Halverson stack ensembles³ should overlap each other by two half-periods. Despite this there will still be reduced weighting of the data at the very beginning and end of a record.

This reduced weight at the beginning has an important benefit in the case of low frequency surveys such as IP and related to the start-up transients in the initial half-periods of a finite record. Owing to the character (long step response decays) of typical IP responses, the first several half-periods after turn-on are usually quite different than the later ones⁴. With normal stacking these differences will produce noticeable errors in the results, with the severity depending on the stack depth. This problem is greatly mitigated by Halverson stacking because of the reduced weighting of the first two half-periods.

² More specifically, the magnitude of the resulting offset in the stacked results is equal to the quarter-period (seconds) times the drift rate in, say, volts per second, and is independent of the stack depth.

³ In general, a record of n half-periods of data is broken in to three or more ensembles so as to allow the calculation of repeatability statistics. So, given a record of 27 half-periods, one could break that into five ensembles, each containing 7 half-periods, which overlap by two half-periods. This allows uniform weighting in the outcome for all but the first two and last two half-periods of the record.

⁴ This is not an issue in data whereby the transmitter has already been operating for some time before the receiver is activated, implying steady-state conditions have been reached.

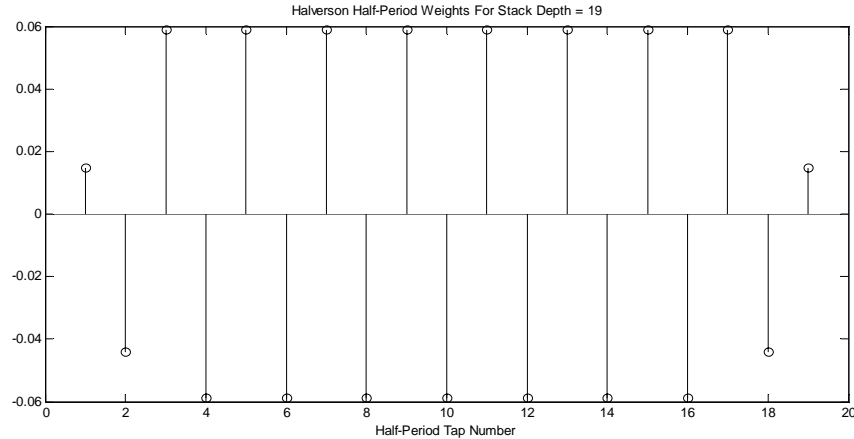


Figure 1 - plot of Halverson stacking half-period weights for a stack depth of 19 half-periods.

Since it is valid to consider stacking as a MA filter operation, it is a relatively trivial matter to calculate the frequency response of stacking operations:

$$H(f) = \sum_{k=0}^{n_s-1} (b_{k+1}) e^{-jkT\pi f} , \text{ where:}$$

- $H()$ - is the complex valued frequency response
- n_s - is the stack depth in half-periods
- b_k - is the k^{th} half-period weight
- j - is the imaginary number ($\sqrt{-1}$)
- T - is the fundamental period of the waveform to be stacked (seconds)
- f - is the frequency of interest (Hz)

Let us compare the frequency responses of normal and Halverson stacking. First we consider IP data with two-second pulse lengths, and a stack depth of thirty-three half-periods (see Figure 2). As expected, stacking rejects the even harmonics and reinforces the odd-harmonics of a periodic waveform. Note in this example how much deeper the even harmonic stopbands are for Halverson stacking, which also enjoys substantially improved low frequency noise rejection.

It bears repeating at this point that Halverson stacking, as compared with the more widely used normal stacking, not only enjoys the critical benefit of rejecting linear drift noise, it also has substantially improved noise rejection in the even harmonic stopbands.

The advantages of Halverson stacking mentioned apply to power-line noise rejection as well, providing the waveform frequency is judiciously selected so that the even harmonics coincide with the power-line noise harmonics⁵. Consider a 12.5 Hz EM waveform and a stack depth of 800 half-periods. The advantages of

⁵ Ideal transmission frequencies are calculated as follows: a) for frequencies greater than the nominal power-line frequency (f_{pwr} in Hz), they should be $(n + 1/2)f_{pwr}$, where n is an integer ≥ 1 , and b) for frequencies less than the nominal power-line frequency, they should be $f_{pwr}/(2n)$, where n is an integer ≥ 1 .

Halverson stacking are obvious as seen in Figure 3. Similar plots for other stack depths show that as the stack depth is increased the 50 Hz⁶ rejection notch increases in width and depth for Halverson stacking, but is not noticeably improved with normal stacking.

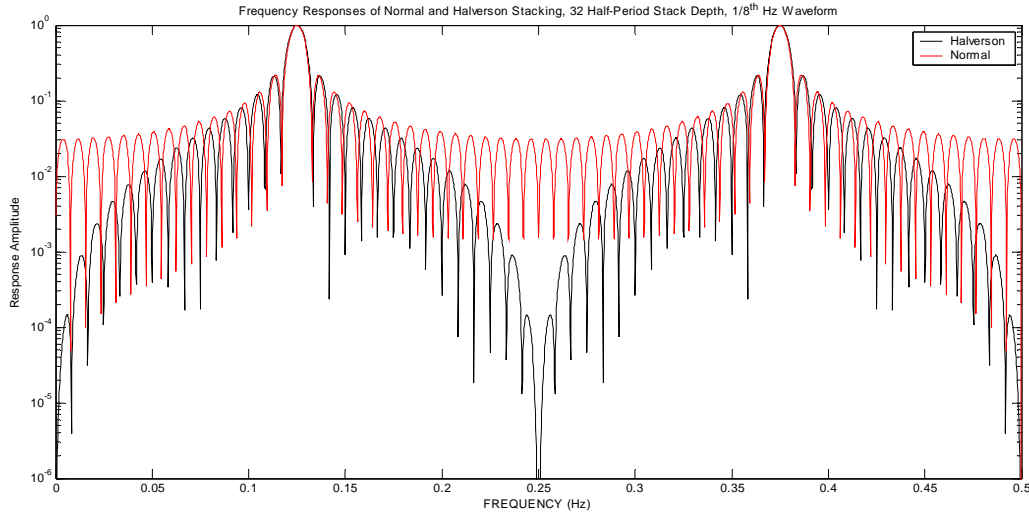


Figure 2 - Comparison of Halverson and "Normal" stacking frequency responses (amplitude) for a typical IP excitation frequency

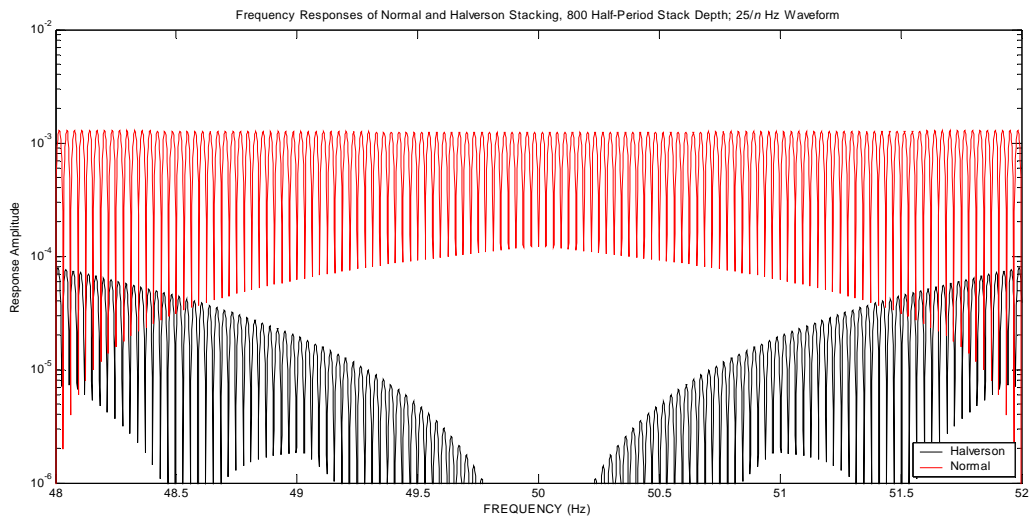


Figure 3 - Halverson and "Normal" stacking frequency responses for a typical EM excitation frequency, in the neighborhood of the nominal power-line fundamental.

⁶ Clearly the same advantages apply to 60 Hz environments (primarily Canada and the US) providing that the transmission frequency is ideally chosen according to the rules in footnote 5.

Tapered Halverson Stacking

Kingman has devised a twist on Halverson stacking whereby linear drift is still removed, but the half-period weights have a much stronger taper to them. This is done by weighting each three half-period group $(\frac{1}{4}, -\frac{1}{2}, \frac{1}{4})$ with values that reflect tapered shapes like Hanning, Kaiser or Hamming windows. To illustrate:

$$\begin{aligned}
 & (\frac{1}{4}, -\frac{1}{2}, \frac{1}{4})/w_1 \\
 + & (-\frac{1}{4}, \frac{1}{2}, -\frac{1}{4})/w_2 \\
 + & (\frac{1}{4}, -\frac{1}{2}, \frac{1}{4})/w_3 \\
 + & (-\frac{1}{4}, \frac{1}{2}, -\frac{1}{4})/w_4 \\
 & \vdots
 \end{aligned}
 , \text{ where the weights } (w_n) \text{ have a tapered and symmetric shape.}$$

Note that because the new stacking filter is based on the Halverson three half-period group, then combining those groups as shown above still affords linear drift removal. The new tapered shape, however, leads to rather spectacular improvements in low frequency noise removal. We illustrate this using a 25 Hz waveform and an **effective stacking depth** of 32 half-periods in Figure 4 below, wherein the taper weights reflect a Hanning window.

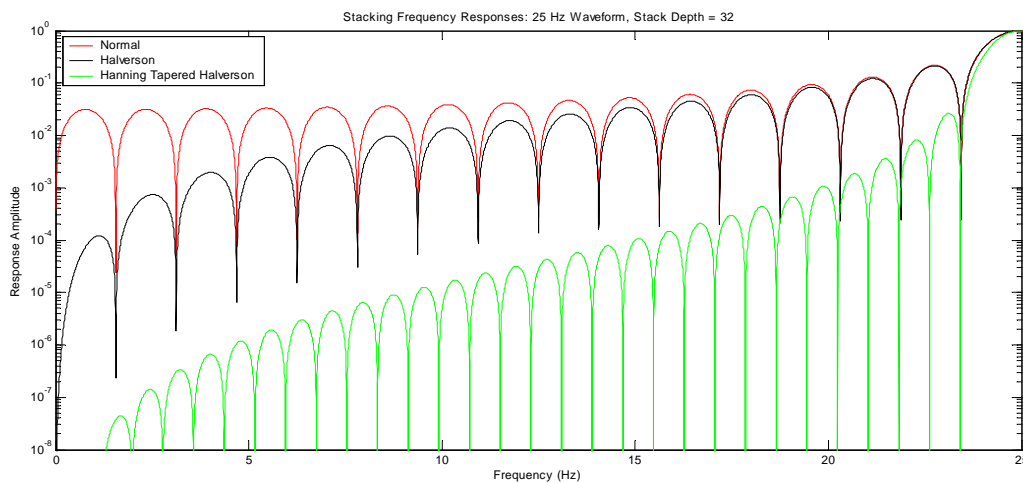


Figure 4 - Halverson, "Normal", and Hanning Tapered Halverson stacking low frequency amplitude responses for a typical EM excitation and effective stack depth of 32 half-periods. Owing to the shape of the tapered filter, this implies an end-to-end depth of 64 half-periods for the Tapered Halverson stack.

The term *effective stacking depth* is used owing to the unusual shape of tapered stacking filters (see Figure 5). Since the half-periods near the beginning and ends of a tapered stack ensemble are down-weighted the stack is not truly averaging the same number of half-periods as for a normal stack (uniformly weighted) of the same length, and the [odd-harmonic] passbands will be wider as a result. We may consider the effective depth of a stacking filter as the sum of the amplitudes of the stacking weights (taps) divided by the amplitude of the weight of largest amplitude, which - perhaps surprisingly - works out to be simply the reciprocal of the amplitude of the amplitude of the weight of largest amplitude⁷.

⁷ This relates to the fact that the stack weights must be normalized to the sum of the magnitudes of the weights in order to have unity gain at the odd-harmonics.

By this standard the passband width of a tapered stacking filter will be roughly the same as that of a normal (uniformly weighted) stack of length equal to the effective stack depth of the tapered filter. This brings us to the useful notion of the ratio of the effective stack depth over the total (end-to-end) stack depth, or Effective Stack Depth Ratio (ESDR) as an indication of the degree of taper. Note that taper stacked ensembles should overlap to a degree that ensures roughly equal weighting of all half-periods in the final averaged estimates. The Hanning taper shape affords perfectly uniform weighting of data if the stack ensembles overlap by exactly one half their length (rounded down) and the stack depth is odd valued.

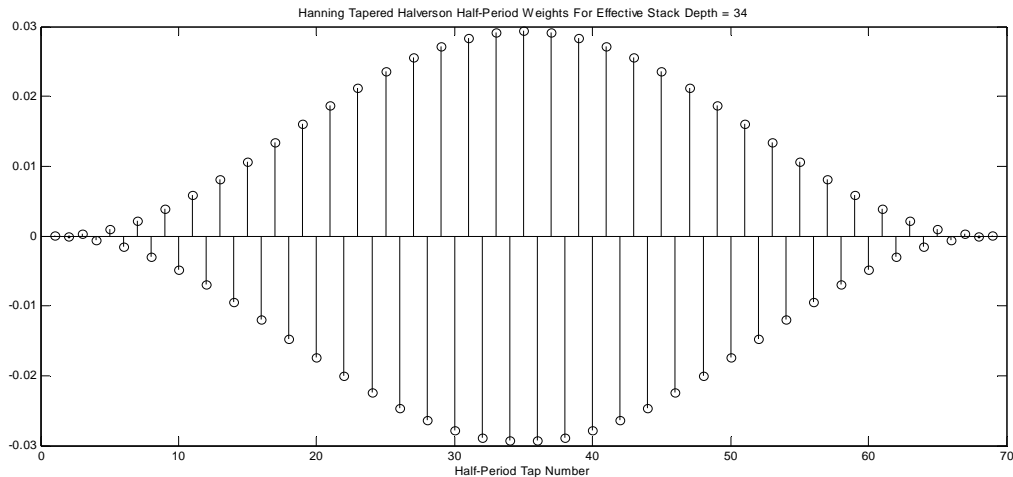


Figure 5 - Half-period weights of a Hanning Tapered Halverson stack filter. The shape and widely varying weights illustrate the need for the term **effective** stack depth, in this case 34 half-periods, which reflect an end-to-end stack depth of 69 half-periods

Mirroring its low-frequency characteristics, Tapered Halverson stacking also provides impressive power-line noise rejection advantages (see Figure 6).

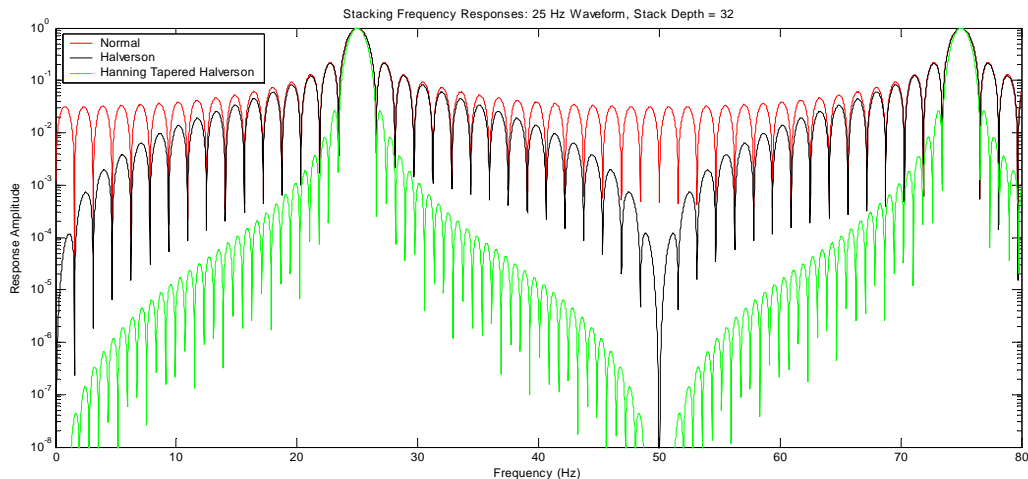


Figure 6 - Halverson, "Normal", and Hanning Tapered Halverson stacking frequency (amplitude) responses for typical EM transmitter frequencies, in the vicinity of the nominal power-line frequency.

It is interesting and important to note that very subtle differences in the weights and shape of the tapered stack filter, differences that may be imperceptible when viewing the filter weights themselves, can show quite large differences in their frequency response characteristics when viewed on a log-amplitude scale. In other words, there are important differences associated with the various taper shapes that should not be ignored. While the Hanning taper is optimal for some applications, a specific Kaiser window⁸ shape may be better for others. Narrow windows with wide tails (small ESDR) provide intriguing results. Below we compare two different and heavily tapered stacking filters with a Hanning taper sharing the same effective stack depth.

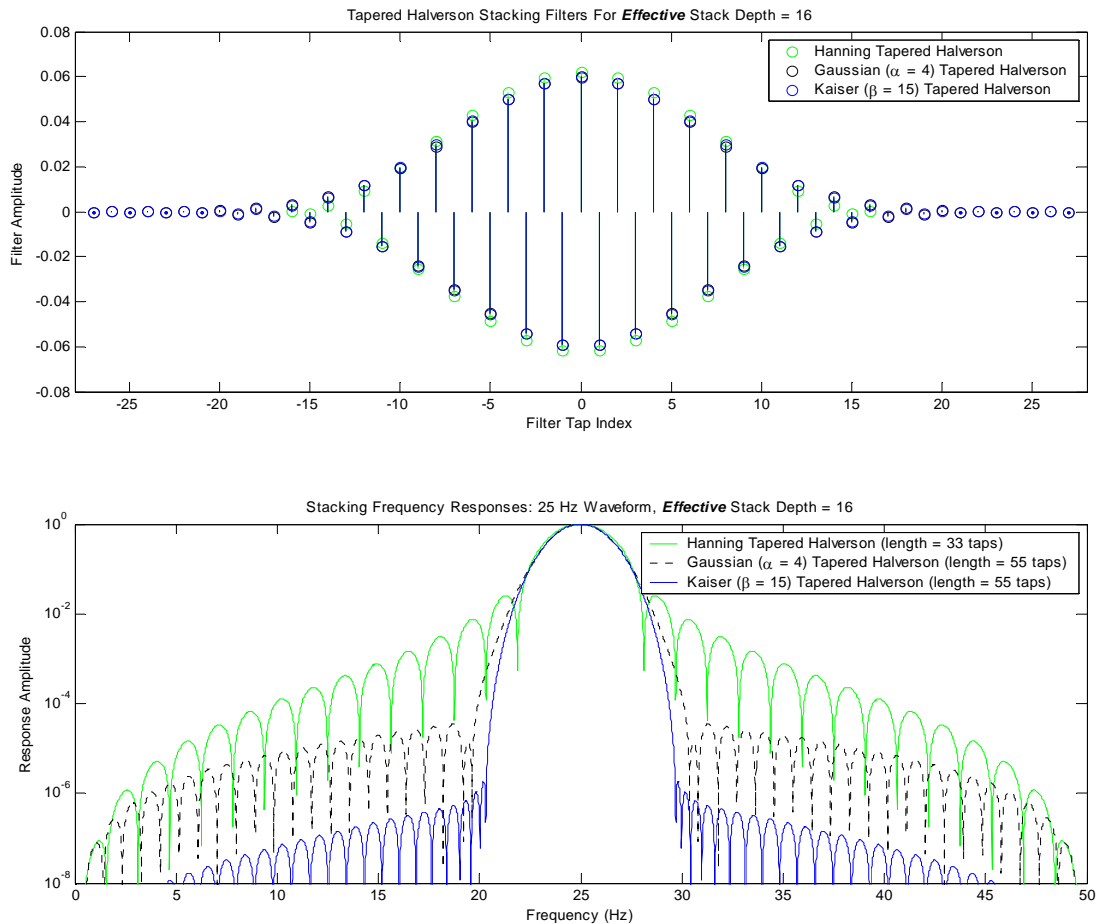


Figure 7 - Halverson Tapered stacking varieties, illustrating a) the advantages of strong tapers (or narrow windows with long tails), and b) advantages of the Kaiser taper shape. All filters shown share roughly the same effective stacking depth - 16 half-periods. However, in order to enjoy that effective stacking depth the lengths of the stacking filters shown are: Hanning: 33 taps, Gaussian($\alpha=4$):55 taps, and Kaiser($\beta=15$): 55 taps. The differences in the Gaussian and Kaiser tapered filter values are visually imperceptible, but nonetheless important as shown in their log-scale frequency responses.

⁸ Kaiser windows form a continuum with shapes determined by a "beta" factor, which dictates the narrowness or Effective Stack Depth Ratio (ESDR) of the window. Kaiser windows with a beta factor greater than roughly 6 are weighted more heavily towards the center and have longer tails than the Hamming window.

Note that for Figure 7, in order to provide the same effective stacking depths (16 half-periods) the Gaussian and Kaiser filters are substantially longer (55 half-periods) than the Hanning (33 half-periods). Further improvements may be had by using even more heavily tapered (but then longer in total length) windows, but these come at the expense of greater total (end-to-end) stack depths. Thus, in general we may say that for a given effective stack depth, more tapered shapes with longer tails are preferred where applicable. For stacking filters of equal lengths, the more tapered windows still enjoy deeper stopband rejection but broader passbands, as shown in Figure 8.

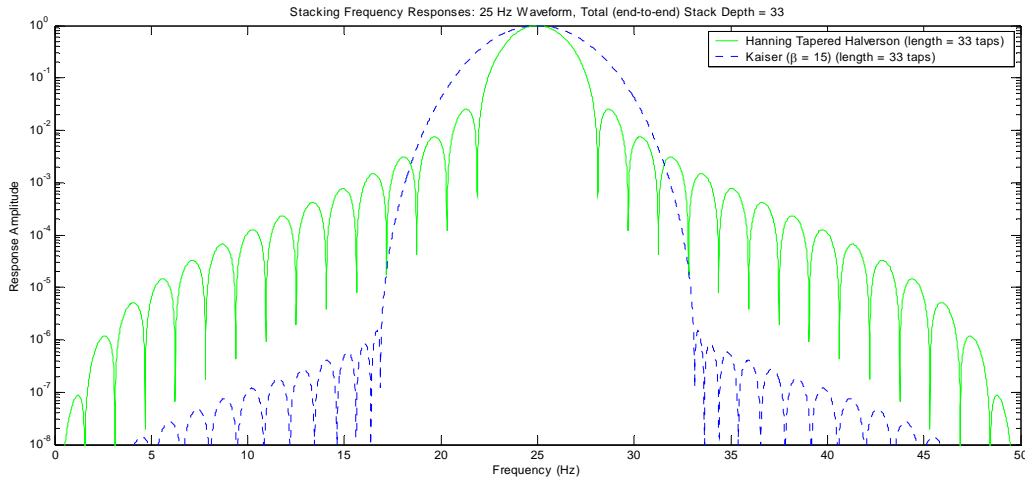


Figure 8 - Identical length Hanning and Kaiser ($\beta=15$) tapered Halverson stacking filters, showing that there is a tradeoff between passband width and stopband depth for a fixed filter length. For equivalent **effective** stack depths, the stronger tapers are advantageous from a strictly noise rejection perspective.

It is interesting to note the difference between the Kaiser Tapered Halverson stacking filter and the straight Kaiser stacking filter without the Halverson linear drift removal character. This is illustrated in Figure 9 below.

Scott MacInnes has given a rather definitive perspective of this Tapered Halverson scheme⁹. Scott noted that the process of ensuring linear drift removal using Halverson's stack kernel $[\frac{1}{4}, -\frac{1}{2}, \frac{1}{4}]$, as noted at the start of this section, equates to a) convolving the tapered window shape of interest with the modulus of that kernel (or a 3-point Hanning window - $[0.5, 1, 0.5]$), b) vector multiplying the convolution result by the "rectification window" $([1, -1, 1, -1, \dots])$, and finally c) properly normalizing/scaling to ensure unity gains at the passbands. Even though it may not be apparent given the fact that the true FIR filter reflected by stacking filters has all those zeros between the half-period weights, we may consider the frequency-domain characteristics of the resulting Tapered Halverson stacking filter to reflect the characteristics of the window (taper) of interest multiplied by (since convolved in the time-domain) the frequency domain characteristics of the 3-point Hanning window.

The use of time-domain windows to mitigate spectral energy leakage is a well-documented and researched subject [Mathworks 2002]. Window shapes commonly used and mentioned include: Hanning, Hamming, Bartlett, Parzen, Blackman and Gaussian. In describing the frequency-domain characteristics of these windows most authors tend to use the terms "mainlobe" and "sidelobe" amplitudes or energy. In our stacking terms these equate to the passbands (narrow as they may be) at the odd-harmonics and stopbands centered at the even harmonics of the filtered periodic waveform.

⁹ Scott also pointed out the advantages of heavily tapered windows in their sidelobe attenuation.

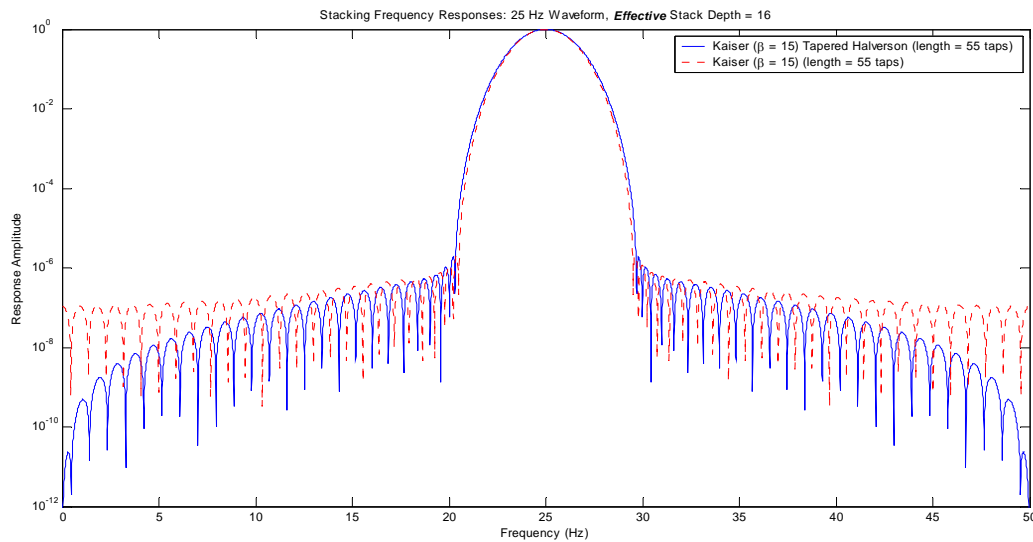


Figure 9 - Two Kaiser window stacking filters, one enjoying the Halverson linear drift removal character, and the other without. On basic visual inspection of the two filters the differences would be imperceptible, however in many common circumstances with low frequency noise that may well be trillions of times greater than the signal of interest, the differences in the results of the two filters will be alarming.

The Kaiser window [Mathworks 2002] allows one to dictate the depth of the stopband or sidelobe attenuation using a β factor and then maximizes the "ratio of the mainlobe energy to the sidelobe energy." Perhaps less well known is the Chebyshev window¹⁰ [Mathworks 2002], which also enjoys the characteristics of allowing one to set the sidelobe amplitude (which is "equi-ripple") and then minimizing the passband or mainlobe width. This speaks directly to our interests in stacking filters, and in general we recommend the Chebyshev Tapered Halverson stacking filter with the trade-off between passband width and stopband depth specified as suggested by the application and noise conditions.

In considering the advantages of Tapered Halverson stacking filters with strong tapers, such as for the Kaiser window with $\beta > 6$, one must bear in mind practical aspects and limitations of having to increase the applied stack depth (i.e. filter length) in order to maintain the desired effective stack depth. Hence Tapered Halverson stacking filters may not be particularly desirable for very low frequency data such as IP because a significant amount of data at the beginning and end of a record is effectively "tossed out" owing to the tapered weighting of the stack filter. A 128 second record of 1/8th Hz IP data spans only 32 half-periods and a Hanning tapered Halverson stack depth of 8 half-periods (17 half-period filter length), appropriately overlapped, would then substantially down-weight roughly one quarter of the measured data.

On the other hand, for a 128 second record of 6.25 Hz EM data, we enjoy 1600 half-periods and can readily employ a strongly tapered filter with few untoward consequences other than increased calculation intensity. For moving platform applications, such as airborne EM, a stacking filter with tails of 64 or more half-periods is not unreasonable providing a commensurate "lead in" to parallel traverse lines is allowed.

Two tapered stacking filters of note that have been published are the *Macnae linear taper* [Macnae 1984] and the *binomial taper* [Lee 2001]. As it turns out, the Macnae taper stack enjoys linear drift cancellation for some depths and taper specifics, but not in general. On the other hand the binomial stack, which is a heavily

¹⁰ Chebyshev IIR digital and analog filter designs are well known, but only related to the Chebyshev window by name, to the authors' knowledge.

tapered stack (ESDR = 0.178), does enjoy linear drift removal for all stack depths (see Figure 10).

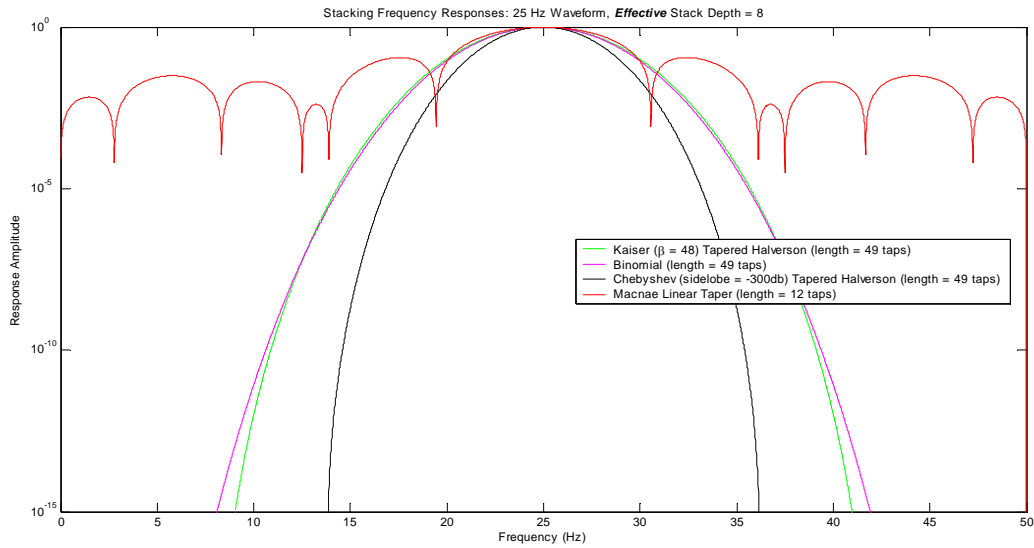


Figure 10 - Comparison of the Macnae linear taper, Binomial taper and Kaiser Tapered Halverson stacking filters for a relatively short effective stack depth. Note that the Kaiser and Binomial filters are considerably longer than the Macnae filter, which is required to achieve equivalent effective stack depths.

Finally, we show Chebyshev Tapered Halverson filter responses for a fixed stack depth (33 half-periods) and a range of sidelobe attenuations or ESDRs. As indicated previously, Chebyshev Tapered Halverson stacking will enjoy maximally (optimum) narrow passbands for a given stopband depth. Hence, by most standards it provides optimized stacking weights from the perspective of its frequency response characteristics.

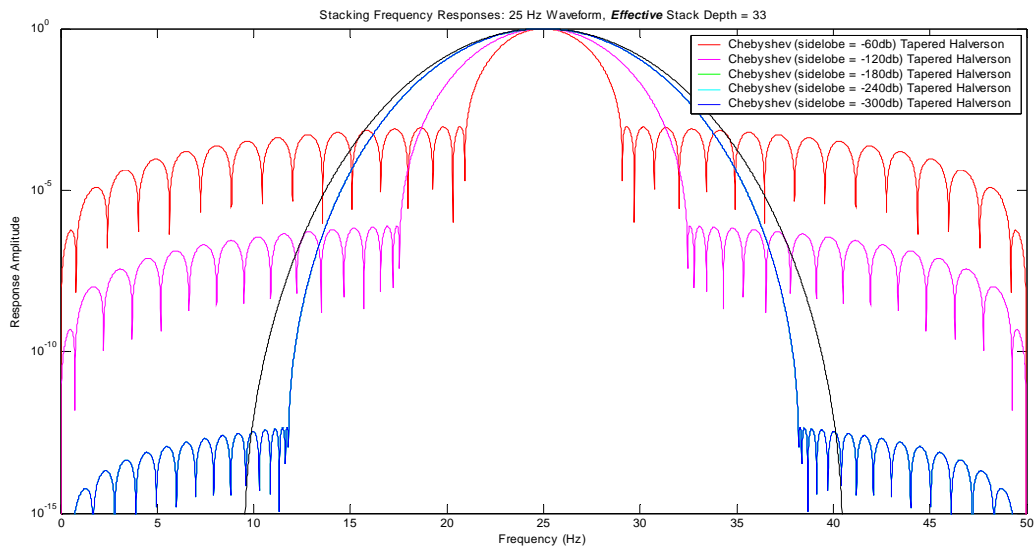


Figure 11 - Various Chebyshev Tapered Halverson stacking responses for a fixed stack depth. Note that the effective stack depth decreases as the stopband attenuation decreases.

SECTION SUMMARY - TAPERED STACKING FILTERS

- Tapered Halverson stacks provide considerably improved noise rejection characteristics as compared with standard Halverson stacking. However, Tapered Halverson stacks come at the expense of reduced data weighting at the beginning and end of a record. This reduced weighting may be extreme in the case of heavily tapered shapes with long tails and so may not be preferred for low frequency data records with few periods.
- The degree of tapering in the Tapered Halverson filter may be chosen as a continuum using either Kaiser or Chebyshev windows with arbitrary β and db factors respectively. For $\beta = 6$ the Kaiser window approximates the Hanning window. At the other extreme, when $\beta = 48$ the Kaiser window approximates the binomial taper of Lee, et. al [Lee 2001]. In many regards the Chebyshev taper provides a flexible and optimized stack design since it provides a somewhat greater effective stacking depth (and hence narrower passband) for a given total stack depth and stopband attenuation.
- Hanning Tapered Halverson stacking filters enjoy particularly deep troughs near DC and the even-harmonics while still maintaining relatively long effective stack depths (ESDR = 0.5).

Localized Periodic Noise

Cultural noise sources other than power-lines such as airports, mines, VLF stations, etc., are of concern as well. If these do not fall sufficiently close to a power-line harmonic, which is normally well attenuated in good stacking designs, then other approaches may be used. It is always possible to play with both the stack depth and the taper shape to make the noise frequency fall on a filter zero (infinite attenuation). For example, consider a troublesome but narrow-band 2345 Hz noise source present in a 25 Hz transmitted wave-train. By slightly changing the taper of a Tapered Halverson filter, in particular using a Kaiser window with a β factor of 8.61 and an effective stack depth of 7 half-periods (full stack depth = 15 half-periods), we locate a filter zero precisely at 2345 Hz as shown in Figure 12 below. So, this process of changing/optimizing the stack depth and taper shape to reject specific noise frequencies can be quite powerful.

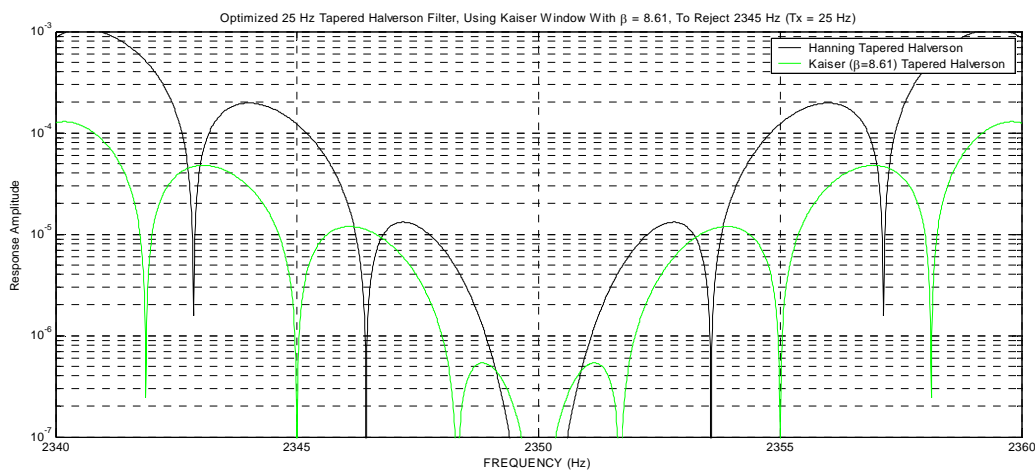


Figure 12 - An optimized Tapered Halverson stacking filter, using a Kaiser window with $\beta = 8.61$, to reject 2345 Hz. The frequency is unimportant - the point is that the flexibility exists to subtly change the filter weights to still enjoy linear drift cancellation and also reject specific frequencies of interest.

Simply forcing stacking frequency response zeros at noise frequencies of concern may not be adequate if the noise sources are too broadband, especially if there is a limit to the maximum useable stack depth as is the case with moving platform surveys. In such cases filtering to supplement the stacking may be required to broaden and deepen the rejection notches. Considering time-domain responses, and presuming that noise is generally not problematic except in the late decay times, we might consider two possible and closely related approaches:

- Filter the time series with an idealized low-pass filter of length that does not greatly impact the shape of the decay¹¹ at late times.
- Idealize the time-gate windows in the decay late times to reject the noise of interest. Note that for most EM fundamental frequencies of interest, this is quite doable for noise frequencies greater than 200 Hz or so, but not easily used against lower frequency noise owing to the limited length of the decay. If the noise is sufficiently high in frequency, an ideally designed time-gate window shape can almost always be discovered that will eliminate it. These shapes should be tapered to provide better attenuation of targeted noise and the optimal shapes are an important design issue just as are stacking weights. Figure 13 below is a useful example.

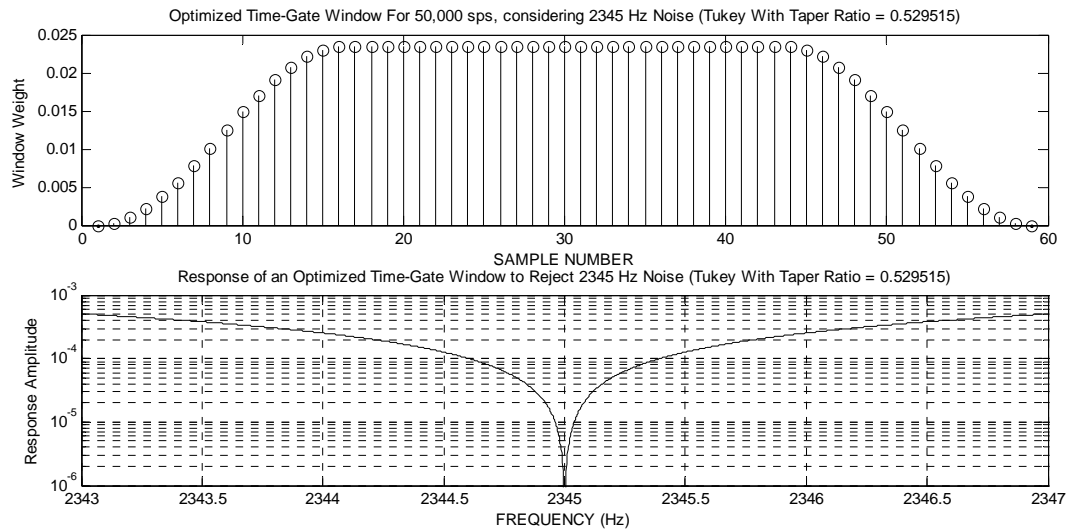


Figure 13 - Illustration of an idealized time-gate window to reject a particular noise frequency. In this case the sampling rate is 50,000 sps and the noise frequency of concern is 2345 Hz. The window width is 59 samples (1.18 milliseconds) or roughly $1/10^{\text{th}}$ of a quarter-period (for a 25Hz transmitted waveform).

So, optimized stacking alone can generally go a long way towards eliminating common noise sources, however additional tricks are often required. Tailoring the processing parameters (stacking filter, time-gate windows, additional filtering) on a case-by-case or region-by-region basis is often worthwhile. A practice of frequently measuring ambient noise during geophysical surveys and adjusting stacking and windowing parameters to optimally reject periodic noise sources could often make the difference in providing high-quality results in otherwise hopelessly cultured areas.

¹¹ It is often easiest to design MA filters that are applied to the response in the frequency domain after stacking.

Moving Platform Applications

As always, bear in mind that like the standard Halverson stacking filter, the Tapered Halverson filter removes linear drift noise. This, plus the low-frequency characteristics of Tapered Halverson stacking can result in startling benefits for some applications such as airborne EM. As noted by others, there can be advantages to measuring or otherwise using the true magnetic field as opposed to its time derivative in EM surveys¹². Considering such a measurement in airborne EM, it is easy to see that the translation of the sensor through the earth's static/DC field can produce low frequency noise of substantial amplitudes, noise that might potentially wipe out the reliability of the late response times in time-domain surveys. The benefits of Tapered Halverson stacking in attenuating such noise are illustrated in a synthesized example below.

Shown in Figure 14 is a reasonable time-varying magnetic field as the receiver bird passes through the earth's field. The influence of a controlled source transmitter is not considered. We take that basic magnetic field time-series and stack it just as if there were a 25 Hz waveform superimposed. Figure 15 shows the resulting half-periods after stacking for effective stack depths of sixteen half-periods. The calculated epsilons indicate the average standard deviations of a given time in the stacked half-period results - they are indications of the noise levels after stacking owing to the movement of the receiver through the earth's static field as previously depicted. Hanning Tapered Halverson stacking provides a reduction in noise roughly 2.5 million times¹³ greater than normal stacking.

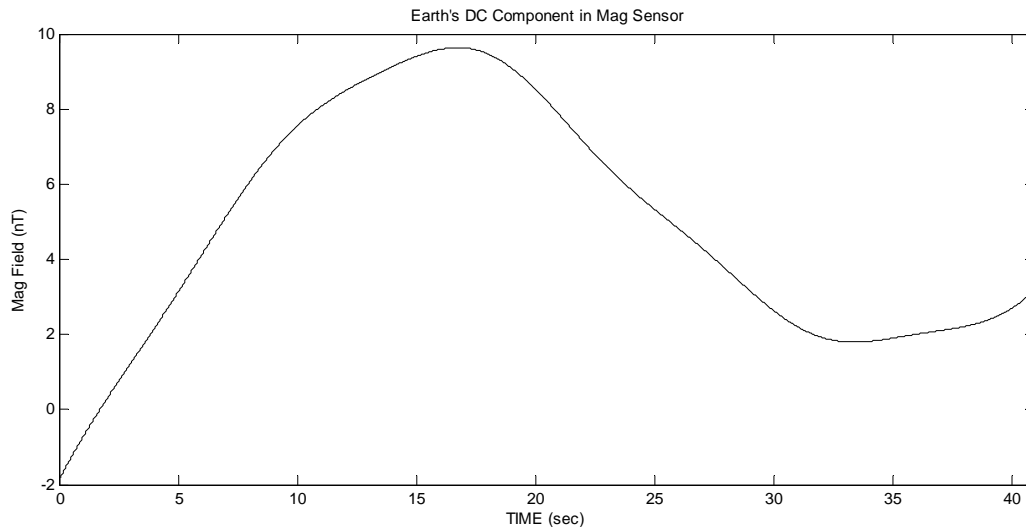


Figure 14 - A synthesized earth's DC magnetic field, with spatial variation translating to a time variation as the sensor travels through the field and across the ground during an airborne EM survey. These data are used to calculate resulting noise after stacking as shown in Figure 15.

¹² The advantages of Tapered Halverson stacking applied to airborne EM data are equally valid for dB/dt measurements, the example is chosen simply because it is easier for most to relate to the magnitude of the noise of concern in terms of the true magnetic field - not its time derivative.

¹³ A disclaimer is warranted here - the level of improvement will be highly dependent on the character of the DC field, or "noise", and the degree to which within a stack ensemble length the noise appears linear. Also, other noise sources, such as bird pitch, yaw, and roll - or higher spatial frequency character in the DC field, may well drop the level of improvement below this particular case. In almost any reasonable case, however, the improvement is expected to be many decades.

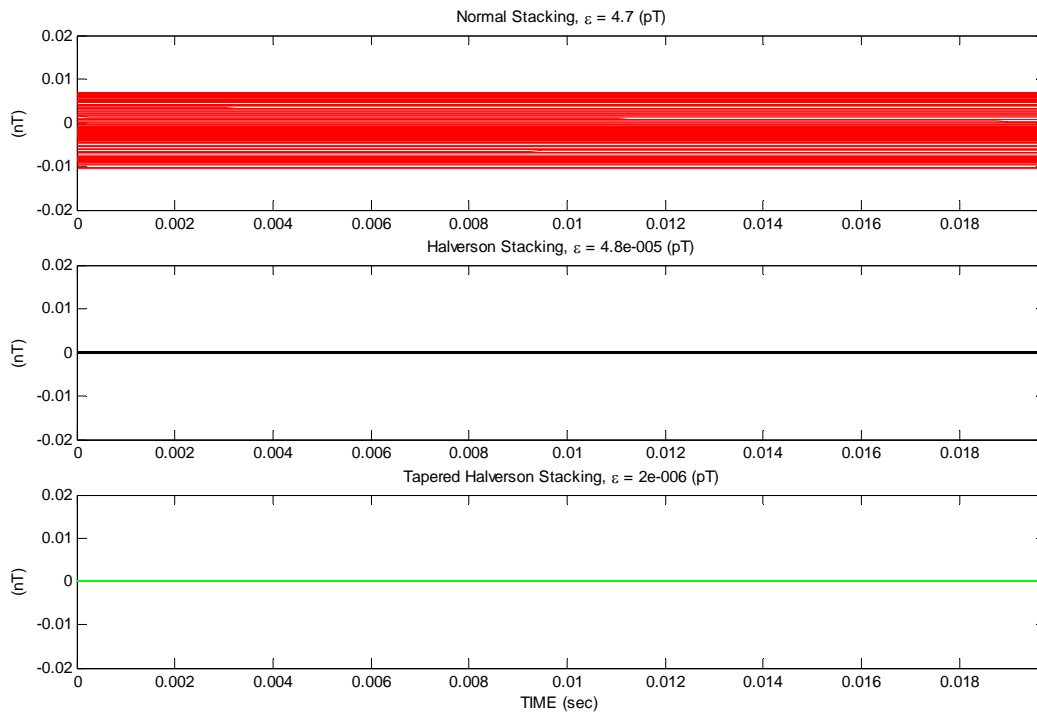


Figure 15 - Half-period results after stacking data shown in Figure 14. The epsilon values indicated are the average standard deviations of points at given/fixed times in the decays and are a result of the low frequency drift noise related to the earth's static field. The presumed transmitted frequency is 25 Hz and the effective stack depth is 16 half-periods.

The possibility of applying either analog or digital high-pass filtering to the signals or data prior to stacking must be considered as a possible solution to this problem, but we find that it is an approach with substantial disadvantages. Let us first consider analog filtering. To enjoy a steep cut-off that is nestled close to the fundamental transmitted frequency, a high-order active filter is needed, which brings with it substantial additive noise. Also, we would strongly prefer linear-phase distortion from the filtering, which is nigh impossible to achieve in analog filters and generally points towards Bessel analog designs which have poor amplitude characteristics in terms of the cut-off steepness. Juggling the choices of amplitude and phase distortion along with additive noise makes analog high-pass filtering quite problematic.

For the digital case acting on sampled data we find that impractically long filters are required to achieve linear phase (requiring symmetric FIR designs) and adequate roll-off. For example, to get acceptable characteristics for a 20 Hz cut-off given 52,000 samples per second, an FIR filter length of roughly 16,000 taps is required. Considering a digital IIR filter, the same distortion difficulties arise as with the analog filter - it is simply next to impossible to achieve a practical filter design that enjoys good distortion characteristics. So we see that the notion of high-pass data filtering prior to stacking while enjoying merit has some fairly severe practical limitations.

Parallel Processing Streams

Consider the idea of forming a parallel processing stream operating on airborne EM data with logarithmically spaced effective stack depths - say 2, 8, 32 and 128 half-periods. These output streams would provide an optimal combination of both high spatial resolution for early time or high frequency results (shorter stack depths) and lower spatial resolution but deeper looking results. The appropriate overlap and tapered stack shape would provide automatic guards against spatial aliasing and thus improve image gridding quality. By also processing separate streams at roughly one-half those mentioned stack depths, repeatability statistics (not often provided with airborne data sets) could be generated based on three smaller overlapping ensembles per larger ensemble.

This, a parallel processing stream of various stack depths using Tapered Halverson stacking, seems an idea of considerable merit. Note again that not only will it provide greatly improved Signal-to-Noise (SNR), but it will also provide an approach that does not suffer compromises between spatial resolution and depth of penetration concerns. It also provides response quality statistics, which are not commonly provided to the authors' knowledge. These statistics will be somewhat biased upward, suggesting slightly larger repeatability errors than actually the case, but nonetheless should be useful. We illustrate the concept diagrammatically below in Figure 16.

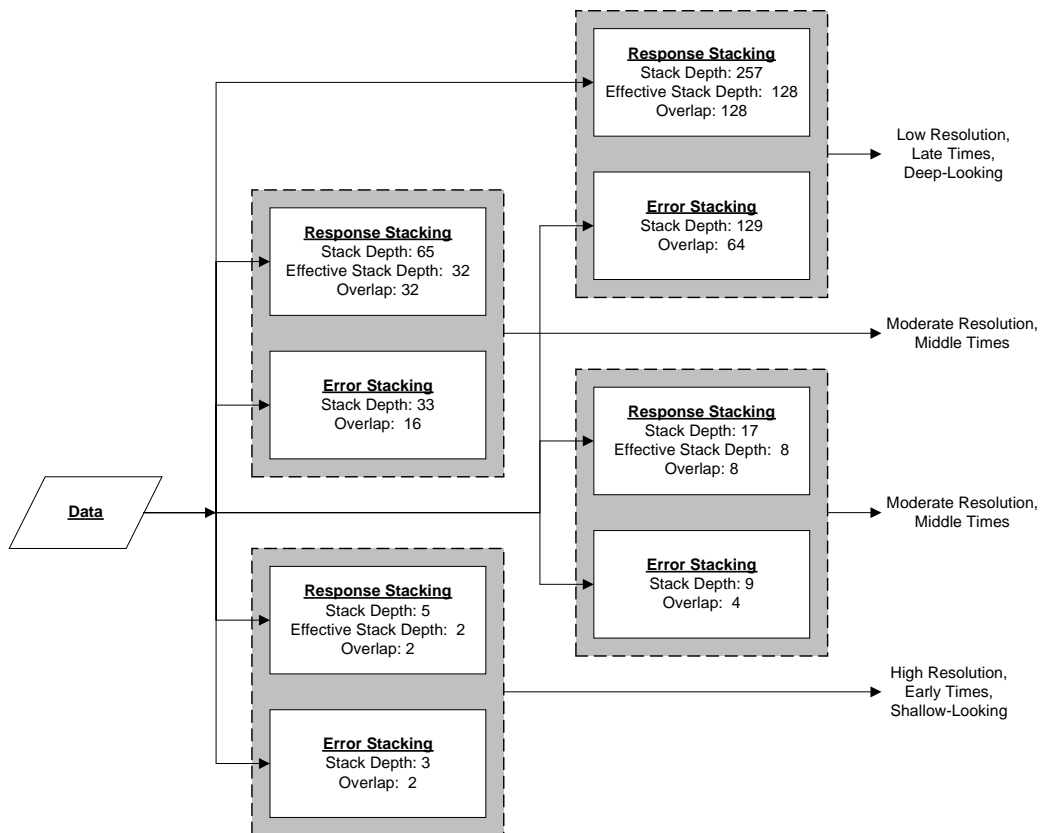


Figure 16 - A suggested processing scheme for moving platform (ground or airborne) data. The purpose is to allow all the advantages of differing stack depths - perhaps allowing deeper depth penetration or IP responses for greater stack depths, while also providing high spatial resolution results. Appropriately sized stack depths to provide repeatability statistics are also associated with each basic output rate.

Conclusions

Tapered Halverson stacking, for carefully chosen taper shapes and transmitter frequencies, enjoys substantial advantages over normal or classical stacking in controlled source electrical geophysics. For cases where there are a small number of periods in a record or data set, Tapered Halverson stacking may be undesirable because of the reduced weighting of the data at the beginning and end of the record. In this case, basic or standard Halverson stacking, or a flattened taper shape such as the Tukey window with narrower passband widths, should be used.

Heavily tapered ($ESDR < 0.25$) window designs can provide spectacular stopband rejection character and the Chebyshev Tapered Halverson filter allows optimizing the passband width versus stopband attenuation for a given total stack depth, although the Hanning taper may still provide better low-frequency noise rejection than some Chebyshev windows.

Tapered Halverson stacking has many special advantages for moving receiver operations such as in airborne EM or roving ground platform type surveys. Parallel stream outputs with logarithmically varying effective stack depths and associated with calculations to allow repeatability statistics, seems an overlooked but quite useful scheme. The possibility has existed for some time, but the advantages of Tapered Halverson stacking in both reducing noise and mitigating special aliasing in the results increases the value of the concept.

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